The Factor Field

by Peter Pike

Introduction

On July 23, 2007 I had a thought. I know the exact day because I wrote about it on my blog. The thought was simple enough to think about, but I would end up spending the next decade of my life wrestling with the various aspects of that thought that would rise up in my life.

The thought began as a simple question. What would it look like if you could actually *see* numbers? I don't mean the digits that we're familiar writing with. I mean, what would it be like to see the characteristics of every single number at a glance, spread out in front of you, as if on a map?

As I began to think of what this would look like, I pictured tangible objects: squares of cloth. My mother used to quilt blankets from time to time, so I imagined a row of blue squares, one after the other. This would be the "ones" column. Next to it, there would be a row of blue squares alternating with black squares, so the blues would appear at every other position. This was the "twos" column. Beside that, two black patches followed by a blue, repeated forever as the "threes."

In this manner, I quickly came to the extent of what I could envision after just three or four columns, and I could only carry it down a few rows before it became a mental blur. But I wanted to see what the shape would look like if I could extend it indefinitely.

These thoughts were in my mind when I went for lunch that summer afternoon. And as I made my way back to the data entry job I held at the time, I realized that I had a tool back on my work computer that would help me envision this task. It was none other than Microsoft Excel. That's right, the spreadsheet program. Who would have guessed you could use it for intense number theory?

So opening Excel, I made a grid by changing the dimensions of each cell to be 20 by 20 pixels. Since I was typing in a value for each square of the grid to represent what I was trying to envision, I decided to fill in the grid with the number 1. I had soon created this graph (the original was actually 100 x 100 in size, but I'm reducing the size a bit to keep it viewable here):

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU	AV	AW	AX	AY	AZ	BA	BB	BC	BD	BE
1		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
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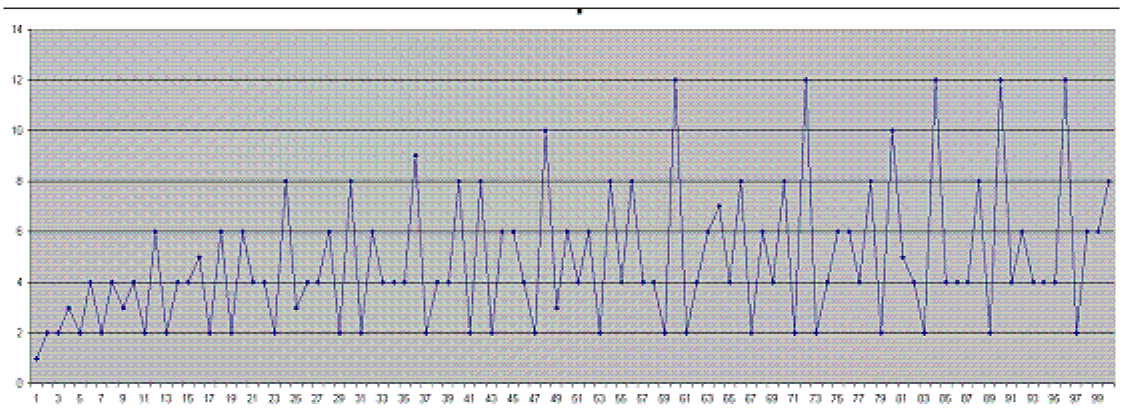
As you can see, I labeled the left-hand column with a row number, and the top row became the header for the columns. If we look at row 1 there is only one entry, and that's under the first column. For row two, there are two entries: 1 and 2. Row three gives us 1 and 3. Row four shows us an entry for 1, 2, and 4.

If you are mathematically inclined, you've almost certainly already spotted the pattern. This graph shows us the factors of every number. As far as I know, the mathematical structure I created here is unique (i.e., no one else has made it). I could be wrong, of course, since I'm a computer programmer, not a professional mathematician. But for now, since I believe I'm the originator of this graph, I'm calling this entire structure the "Factor Field" since it represents the factors of each number.

Now since I had used "1" as my input value, I was able to do some calculations. At the end of each row, I added a formula that would add up all the values in the row (which I'm calling the "Factor Field Row

Value"). This value is equivalent to the number of factors for each row. I then created a graph in Excel to represent it:



And this is what I teased my blog audience with that fateful day in 2007.

The method I use surreptitiously gives us an added benefit in that we can easily come up with a definition for prime numbers that is more complete than normal definitions are. See, normally we're told that a prime number is a number that cannot be evenly divided by any other number but itself and 1. But of course, under certain definitions of the words used, the number 1 also is only divisible by itself and 1. However, 1 is not, and can never be, prime. There are many reasons why it's not prime, but the best reason is that if we were to make 1 prime then it would violate the Unique Prime Factorization Theorem.

That theorem states that each number greater than one is either prime or else is a composite number composed of a unique set of prime numbers. The word "unique" is critical, because what we're saying is that given a certain "recipe" of prime numbers, we will get one, and only one, result. Take the number 24 for example. 24 is the same as two cubed times three. Now, you can arrange the multiplication any way that you want. You can start with 2 x 3 to get 6, then multiply that by 2 for 12, then again by 2 to get 24. Or you can start with 2 x 2 to get 4. Multiply it by 3 to get 12, then 2 to get 24.

But what you can't do is multiply by 2, say, five times to get to 24. The number 24 must have three multiples of 2 and one multiple of 3 in its factorization. That's the unique factorization it has.

But what if 1 is considered to be a prime number? In that case, you could have 2 cubed times 3 times 1. Or 2 cubed times 3 times 1 squared. Or 1 cubed. Or 1 to the 57th power. All of these would equal 24 again. So if 1 is a prime number, then that means that there are an infinite number of ways to factor a composite number, not a single unique way. So 1 clearly can't be considered prime if the Unique Factorization Theorem is to stand.

But as I said, using the Factor Field Row Value to define a prime number, we can automatically exclude 1. That definition of "prime" is: "A prime number is that which has a Factor Field Row Value of exactly 2." The Factor Field Row Value at row 1 only has a value of 1, so the integer represented by that Factor Row (the integer 1) is not prime. However, the Factor Field Row Value at row 7, which represents the integer 7, has a Factor Field Row Value of 2. Therefore, the integer represented by row 7 is prime.

Equally quickly, we have a definition for composite numbers. “Numbers that are composite have a Factor Field Row Value greater than 2.” For example, the aforementioned number 24, represented as row 24 on the Factor Field, has a Factor Field Row Value of 8. (The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.) Therefore, the integer represented by the Factor Field at row 24 (the integer 24) is a composite number.

Now, you may have noticed the somewhat clinical language being used there, viz. “The integer represented by the Factor Field at Row” and so forth. That is intentional at the offset because I want to assure skeptics that there is a difference between the abstract numbers and the visual representation of them. However, from this point on, I believe that the congruence between the graph and the abstract numbers is so sound that we can simply cut through the clunky language and say, “The graph shows row 7 only has 2 factors, therefore 7 is prime.” It’s more natural to talk this way, but please keep in mind that graphs are mere representations of the abstract numbers, and not really the numbers themselves.

To continue with some of the history here, my first Factor Field was typed in manually. This quickly became far too inconvenient, so I began to experiment with different Excel macros, culminating in creating a Factor Field that filled the entire Excel 2003 spreadsheet: 256 columns wide and 65,536 rows deep! But even macros proved to be too time consuming and too clunky, so eventually I wrote a script using the VBScript language that would create a Factor Field for me.

This gets me to the official definition of the Factor Field. As I stated above, I’m a computer programmer first, not a mathematician. In fact, I tend to do math by the scientific method, wherein I observe something, form a hypothesis, and then test it. If I get enough successful tests, then I form a hypothesis. Nevertheless, I understand that this process is not rigorous enough to be mathematical proof, so having formed a hypothesis I generally seek to try to prove the point rigorously using the amount of mathematical reasoning I do know. Sometimes, I do not have the full language needed to fully state something in mathematical language, however, and this is one of those times.

So for the record, I believe that the following pseudo-code is the basis of the Factor Field, where the loops carry on to Infinity:

```
For x = 1 to Infinity
    For y = 1 to Infinity skipping by x
        Point (x,y) is a member of the Factor Field
    Next y
Next x
```

This structure above is the definition of the complete Factor Field but I have no idea how to represent that mathematically. Additionally, a computer cannot generate the entire structure, so I place limitations on it when generating code. To fully explain the graph, perhaps it will help to show the code in VBScript.

Should you wish to view this yourself, all you need is a computer with Windows and Excel installed, and the knowledge of how to run VBScripts, and these few lines will make your own Factor Field for you:

```
set objExcel = CreateObject("Excel.Application")
set objWorkbook = objExcel.Workbooks.Add()
set objWorksheet = objWorkbook.Worksheets(1)
```

```
objExcel.Visible = True

MaxWidth = 100
MaxDepth = 150

for x = 1 to MaxWidth
  for y = x to MaxDepth step x
        objExcel.Cells(y,x).Interior.ColorIndex = 1
  next
next

wscript.Echo "Done."
```

The first four lines can be safely ignored for now, as they are just VBScripts way of opening Excel and preparing it to show everything. After that, I've given some constraints. The MaxWidth will be 100 cells wide, and the MaxDepth will be 150. When run, this will show what the Factor Field would look like for the 100 x 150 range. Additionally, the command

```
        objExcel.Cells(y,x).Interior.ColorIndex = 1
```

just fills the cell located at (y,x) with a black background. (It's somewhat annoying, but Excel references cells with the row first, then the column, so the x and y coordinates are "backwards" from math here.)

The important part that we need to look at are the loops. The "for" keyword in VBScript instructs the computer to repeat a section (everything in the code up to the keyword "next") until the value of the variable linked to the loop reaches a specific termination value. So for those who are neither programmers nor mathematically inclined, I'll take this next bit a little slow.

The loop begins with the keyword "for" and then we have a variable, **x**. This variable functions similarly to the way that variables function in Algebra—that is, the **x** will represent some other value and that value can change. However, unlike Algebra, the value of **x** can never be *unknown* by the computer. That's why we have to start **x** off with an initial value, in this case we start with **x = 1**.

So now we've got the start of the loop with the initial value of 1, but we need to know when the loop should stop. And that's found by the value listed after the keyword "to", which in this case is the value of the variable **MaxWidth**. **MaxWidth**, as you can see earlier in the code, has been set to 100 for the purposes of this demonstration.

So the loop we have begins at 1 and it will go to 100. And what does it do while it loops? It executes all the instructions between it and the "next" keyword. But what are those instructions? They're *another* "for" loop! That's right, you can stack "for" loops inside each other in a process called "nesting."

What is that second "for" loop doing? Well, this time we've got a different variable, the variable **y**. Once again, it has to begin by being set to a specific starting value. Just as in the outer loop we had to specify that **x = 1** was the start of the loop, here we see that the value of **y** is set: **y = x**. Then, we see that this loop will go to **MaxDepth**, which for this demonstration we have set at the value of 150. There is one final section to the code, the "step" value, which we will discuss in a little bit.

For now, let us think about what happens. The outer loop has **x** going from 1 to 100. The inner loop has **y** going from **x** to 150. So what happens? Let's look at it step by step:

1. The computer begins the outer loop, assigning the value of 1 to the variable **x**.
2. The computer then begins the inner loop, assigning the value of **x**, which is 1, to the variable **y**.
3. The computer then does all the instructions in the inner loop, and begins by printing a value at column **x**, row **y**.
4. The variable **y** is now incremented (with rules determined by the "step" feature we will discuss shortly), and the computer loops back to instruction 3 until **y** reaches the value of **MaxDepth**.
5. When **y** is greater than **MaxDepth**, the loop ends and the computer returns to the outer loop.
6. The only command after the inner loop is the outer loop's own "next", so the computer increments **x**. Now, **x** is set to 2.
7. The computer then begins the inner loop again, assigning the value of **x**, which is now 2, to the variable **y**.
8. Steps 3 – 6 are repeated.

This continues until **x** is greater than **MaxWidth**. At that point, the outer loop is finished, and the program continues to the next set of instructions after the loops, which in this case just tells the computer to pop up a window saying it's done.

In a nested loop, the inner loop will *never* be run unless the outer loop is run. The inner loop is dependent upon the outer loop for its existence. While this may not be immediately obvious, there are a couple of examples of such constructs in our normal, everyday life. For example, think of a clock. In the United States, we use a 12 hour clock (unless you're in the military, which uses the 24 hour clock). This means that we go through all 12 hours twice a day. Furthermore, each hour is divided into 60 minutes, and each minute into 60 seconds. If we wanted to describe this behavior in terms of loops, we could set it up the following way:

```
for Day = 1 to 2
  for Hour = 0 to 11
    for Minute = 0 to 59
      for Second = 0 to 59
        Time = Hour:Minute:Second Day
      next
    next
  next
next
```

So, the above structure works like this. First, we look at the part of the day. If Day = 1, it's AM. If Day = 2, it's PM. (Of course, this doesn't work *exactly* the way it should, because AM and PM both start at 12:00, not at 0:00—but that fact doesn't change how the loop structure can be understood.) After accounting for AM and PM, we iterate through the hours, which go from 0 to 11. Then we get the minutes, which go from 0 to 59. Then, the seconds, which likewise go from 0 to 59.

So that is how nested loops work. But, if you recall, the loop used to create the Factor Field has the keyword "step". Many times when we go through loops, we want them to increment one at a time. Thus, in the example involving time, we wanted the seconds to go from 0 to 1 to 2, etc. and likewise for the minutes. Everything increased by one each time through the loop. But there will be times when we do not want to count by 1, and VBScript allows you to change the frequency of your count by adding the "step" keyword.

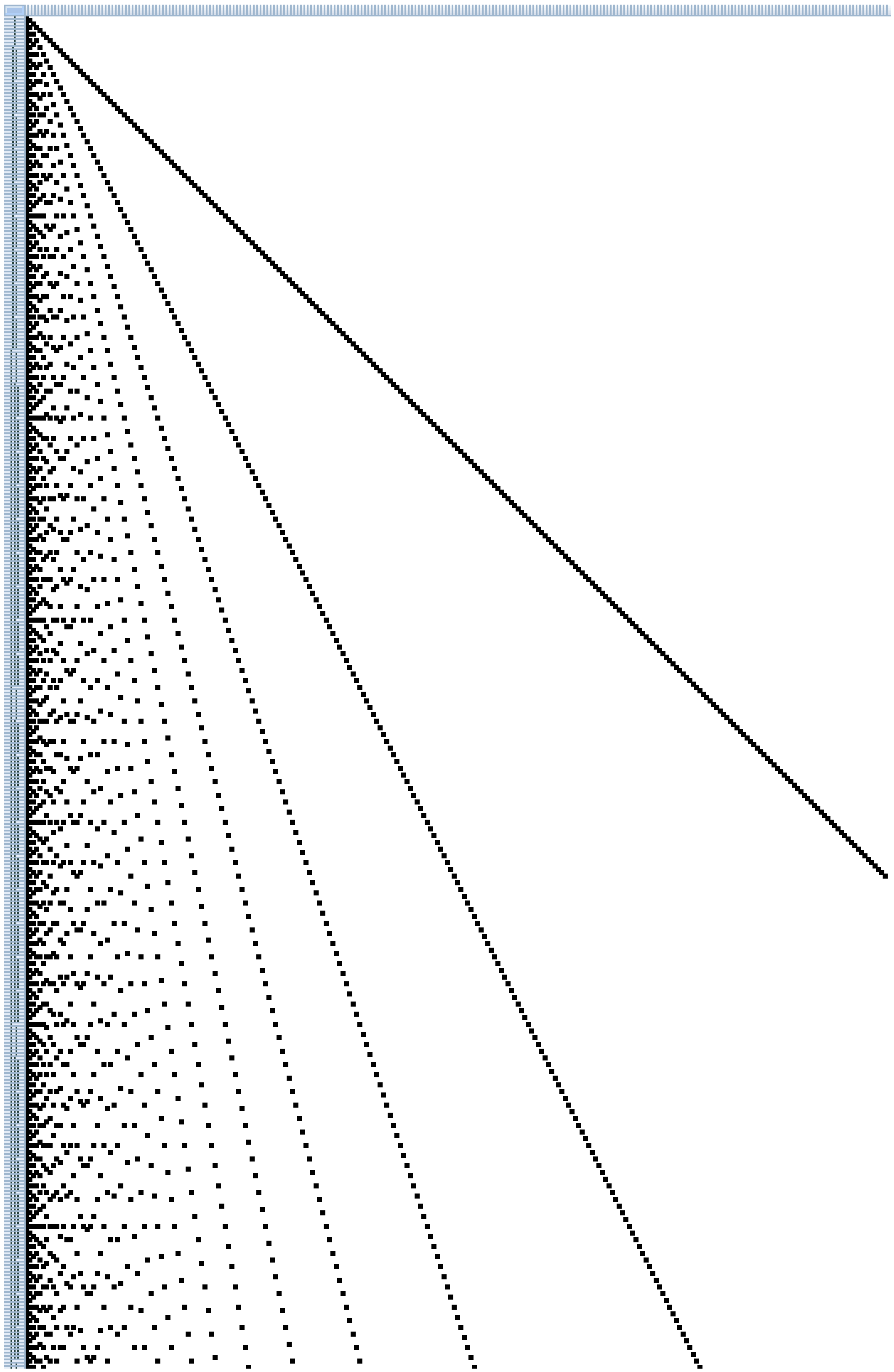
What the “step” keyword does is increment by the value following “step”. So, looking at the relevant code again, we see:

```
for y = x to MaxDepth step x
```

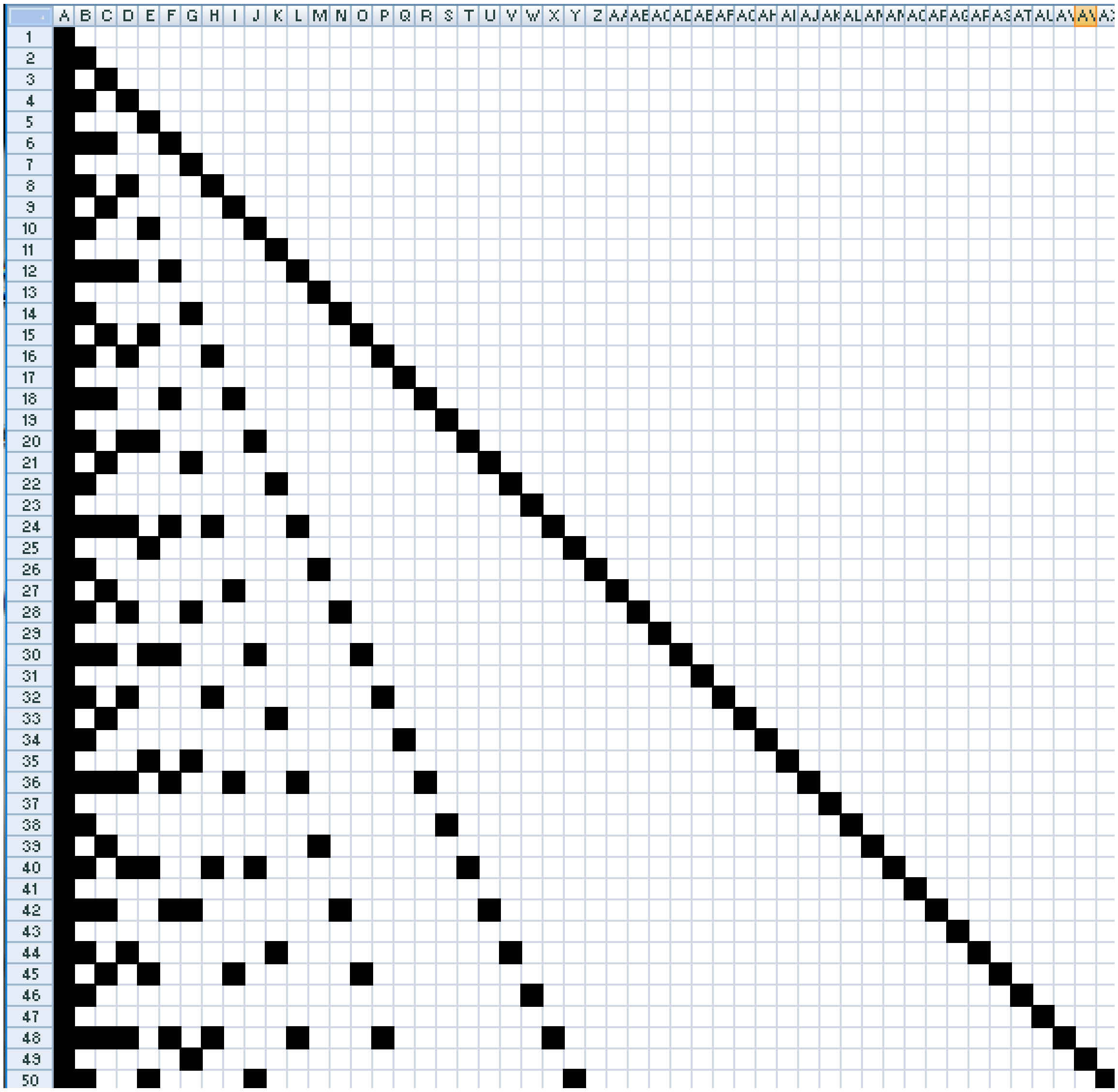
In this **y** loop, not only does the loop start counting from whatever the value of **x** is, but the count for **y** varies according to **x** too. Therefore, the first time through, **x** is set to 1. This means the **y** loop will go from 1 to 150 (the value of **MaxDepth**) with a “step” of 1, which is to say it adds 1 to the value each time through. But the second time through the loop, **x** is set to 2. Now, **y** goes from 2 to 150 with a “step” of 2, which is to say the computer adds *2* to the value of **y** each iteration. The third time, **x** is 3, so the “step” is 3, etc.

The result is that the values for **y** when **x = 1** are the set of numbers {1, 2, 3 … 149, 150}. When **x = 2**, however, the values for **y** are {2, 4, 6, 8 … 148, 150}. When **x** = 3, the values are {3, 6, 9 … 147, 150}. And so on. Because of this feature, the above nested loops are able to output the Factor Field.

So, now knowing that, let’s look at the beginning of the Factor Field, using a width of 256 and a depth of 15,000 (note, even compressing the graph to 10% in Excel, as I did here, will not show all 15,000 rows):



Let us examine the beginning in more detail by zooming in to the top 50 x 50 square.

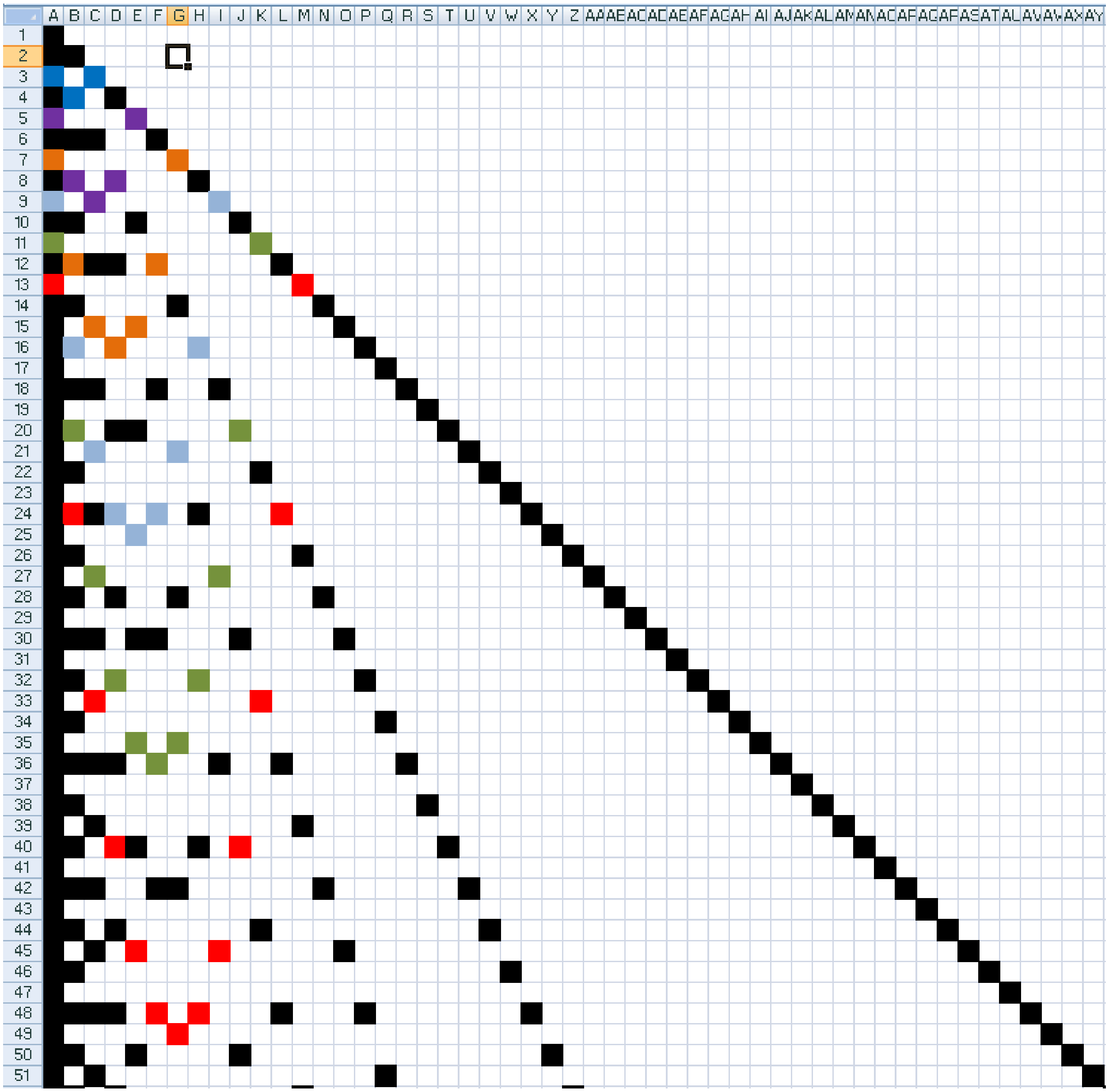


Now the first thing I notice when looking at this is the slopes of the lines. We have the original slope leading from (1,1) down to the right to (50,50)—and note that we are now using the familiar (x,y) coordinate scheme instead of Excel's "backwards" coordinates from here on out. This line is unbroken, but the next line that goes from (1,2) – (25,50) and only appears at every other y value. The next line goes from (1,3) to (16,48), and drops down three y spots for every value of x we increment.

In other words, the diagonals are mimicking the "step" skipping that we had above to generate the original graph. In fact, we could rearrange the loop to print out the graph on a diagonal instead of column-by-column and it would still work.

Parabolas

But one of the more intriguing aspects in the existence of parabolas in the graph. See here:



Here, I've color-coded the various parabolas. Now, I find a few things interesting about this. First of all, it's very intriguing that every single parabola is "anchored" onto the (1,1) to (1,50) column, and the anchor occurs every other row. This pattern indicates that (1,1) is itself the origin of the pattern, meaning if it was actually possible to extend it out it would have been a parabola too.

The next interesting bit is the "chevron" that points to the origin (a "chevron" looks a little like a v). If we look at the point, we see the following values: (2,4), (3,9), (4,16), (5,25), (6,36), (7,49). In each instance, the **y** value is the square of the **x** value: $2^2 = 4$, $3^2 = 9$, 4^2=16, etc. So this parabola actually points to the squares. This is also why I made my earlier statement that (1,1) was part of this process, since $1^2 = 1$.

In other words, just by looking at the factor field, it is possible to *see* every single perfect square. And not only that, but the wings of the parabola are also explained. I'll only take the final parabola for now, with the origin of (7,49). If we look at the row above it, we are at 48. The two factors on 48 that make up the wings are 6 and 8. Notice also that 6 x 8 = 48, so those two factors multiply together to give the row number they're on too.

Not only that, but years ago I had realized by playing with a calculator that the square of a number is equal to one less than that number multiplied by one more than that number, plus one. Now, that's a mouthful to read. Symbolically it's this: $n^2 = (n - 1)(n + 1) + 1$

If you factor $(n - 1)(n + 1)$, you get:

$n^2 = \mathbf{n^2 - n + n - 1} + 1$

The $-n + n$ cancel each other out, as does the -1 and the 1, so you're left with

$n^2 = n^2$

which proves the conjecture true. But now, we can carry that out further. The next set up the "wings" of the parabola is at row 45. There we see columns 5 and 9. Once again, 5 x 9 = 45. And this time we can look at the following pattern: $n^2 = (n - 2)(n + 2) + 4$. The 4 is because we have to add 4 rows to 45 to get back to row 49. Once again, factoring that equation, we have $-2n + 2n$ and -2 + 2 cancel each other out, leaving $n^2 = n^2$.

This continues on up the line. At row 40 it's 4 and 10; row 33 is 3 and 11, row 24 is 2 and 12, and finally row 13 is 13 and 1. Meanwhile, the integer added at the end increases from 1, to 4, to 9, to 16, to 25, to 36.

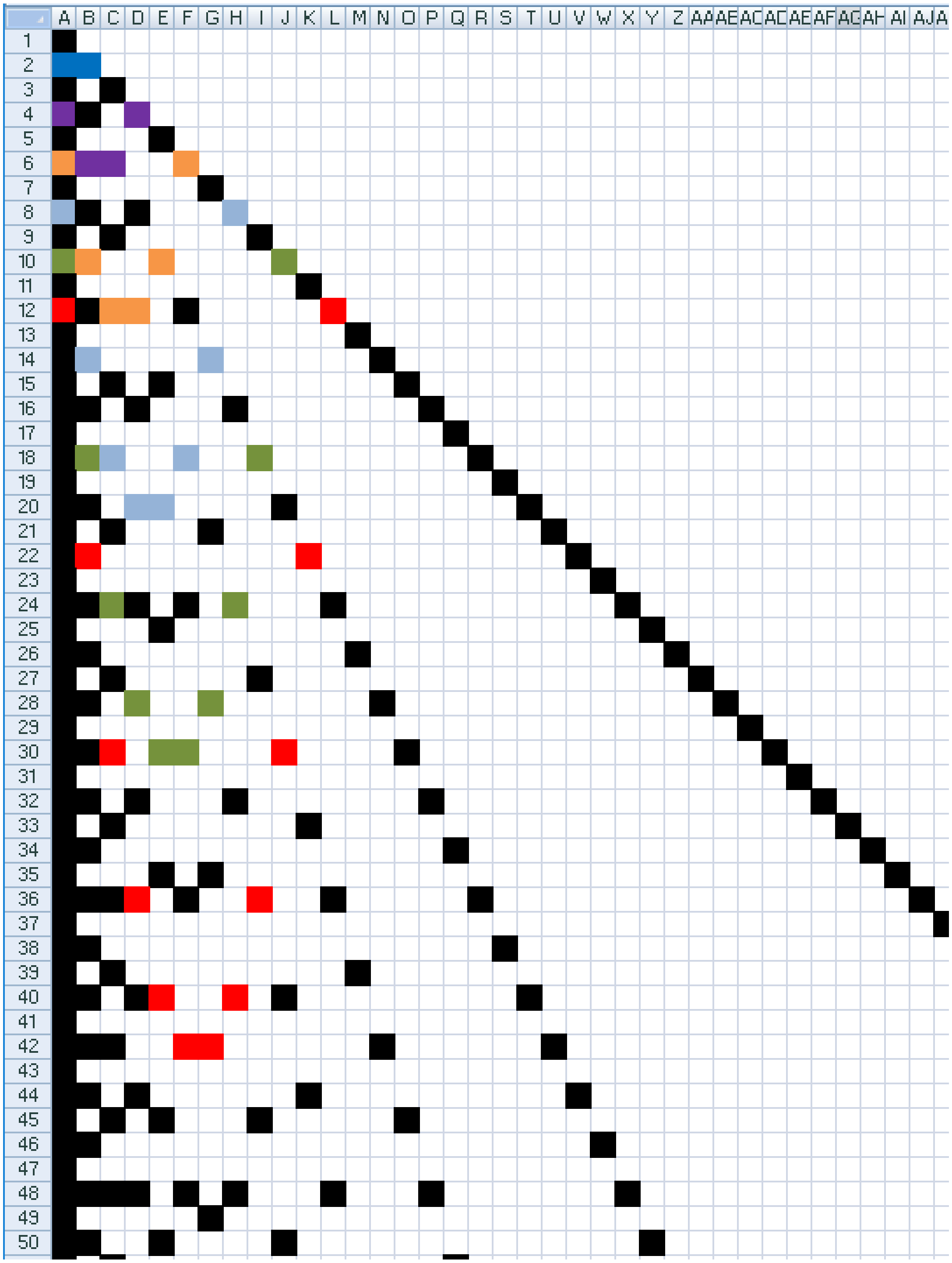
Wait…those numbers also look familiar, don't they? Sure enough, we are going through the squares once again. In other words, we can generalize even further. Not only is it the case that $n^2 = (n - 1)(n + 1) + 1$, but it's actually $n^2 = (n - a)(n + a) + a^2$. And we can prove that by factoring:

$n^2 = n^2 - na + na - a^2 + a^2$.

After cancelling, we once again get our familiar: $n^2 = n^2$

Now, the above is not exactly earth-shattering news. But what's cool is that I was able to discover this *just by looking at a graph*. The Factor Field itself was hiding this data. No calculation is needed to find it: just look at the graph and get your answer!

But there's more. You may have noticed that there is another set of parabolas in between these parabolas I've dealt with so far. Allow me to illustrate them:



The first thing we see with this is that the “anchor” bits in the (1,1) column match the “missing” bits in the first series. Furthermore, this parabola exists nearly exactly halfway between the parabolas formed by the “chevron” at the origin. Could this therefore somehow represent the squares halfway between the integers? That is, could the parabola that has the origin on row 42, at columns 6 and 7 (again 6 x 7 = 42) somehow be the square of 6.5?

Well, if we square 6.5 we see that sure enough it is almost equal to 42. The exact answer is: 42.25. We can also see whether this has a similar pattern to what we saw earlier—that is, $n^2 = (n-1)(n+1) + 1$. In this instance, we are examining not n^2 but $(n + 0.5)^2$. That is, we're looking at the square of the number halfway between various *n*s. If it is to hold to the same pattern, where we multiply the two factors immediately before the (missing from the graph, because it's a fraction) origin, and then add the difference between that result and the origin, we should see:

$(n + 0.5)^2 = n(n + 1) + 0.25$

And we can prove this:

1. $(n + 0.5)^2 = n(n + 1) + 0.25$
2. $(n + 0.5)(n + 0.5) = n^2 + n + 0.25$
3. $n^2 + 0.5n + 0.5n + 0.25 = n^2 + n + 0.25$
4. $n^2 + n + 0.25 = n^2 + n + 0.25$

So the math works here. Furthermore, we can see that it works if we take row 40, where the factors are 5 and 8 (again, 5 x 8 = 40). Here, the pattern becomes:

1. $(n + 0.5)^2 = (n-1)(n + 2) + 0.25 + 2$
2. $n^2 + n + 0.25 = n^2 +2n -n -2 + 0.25 + 2$
3. $n^2 + n + 0.25 = n^2 + n + 0.25$

So once again, equivalency is found. And the next up the list is at row 36 with the factors of 4 and 9, etc. This time, it's $(n + 0.5)^2 = (n-2)(n + 3) +0.25 + 6$, which you can factor yourself.

The generalized pattern is: $(n + 0.5)^2 = (n -a)(n +a + 1) + 0.25 +(a^2 + a)$, as follows:

1. $(n + 0.5)^2 = (n -a)(n +a + 1) + 0.25 +(a^2 + a)$
2. $n^2 + n + 0.25 = n^2 + na + n - na - a^2 - a+ (a^2 + a)$
3. $0 = - a^2 - a+ (a^2 + a)$ [After cancelling terms]
4. $a^2 + a = a^2 + a$

And to check the general pattern:

When $a = 0$, you get:

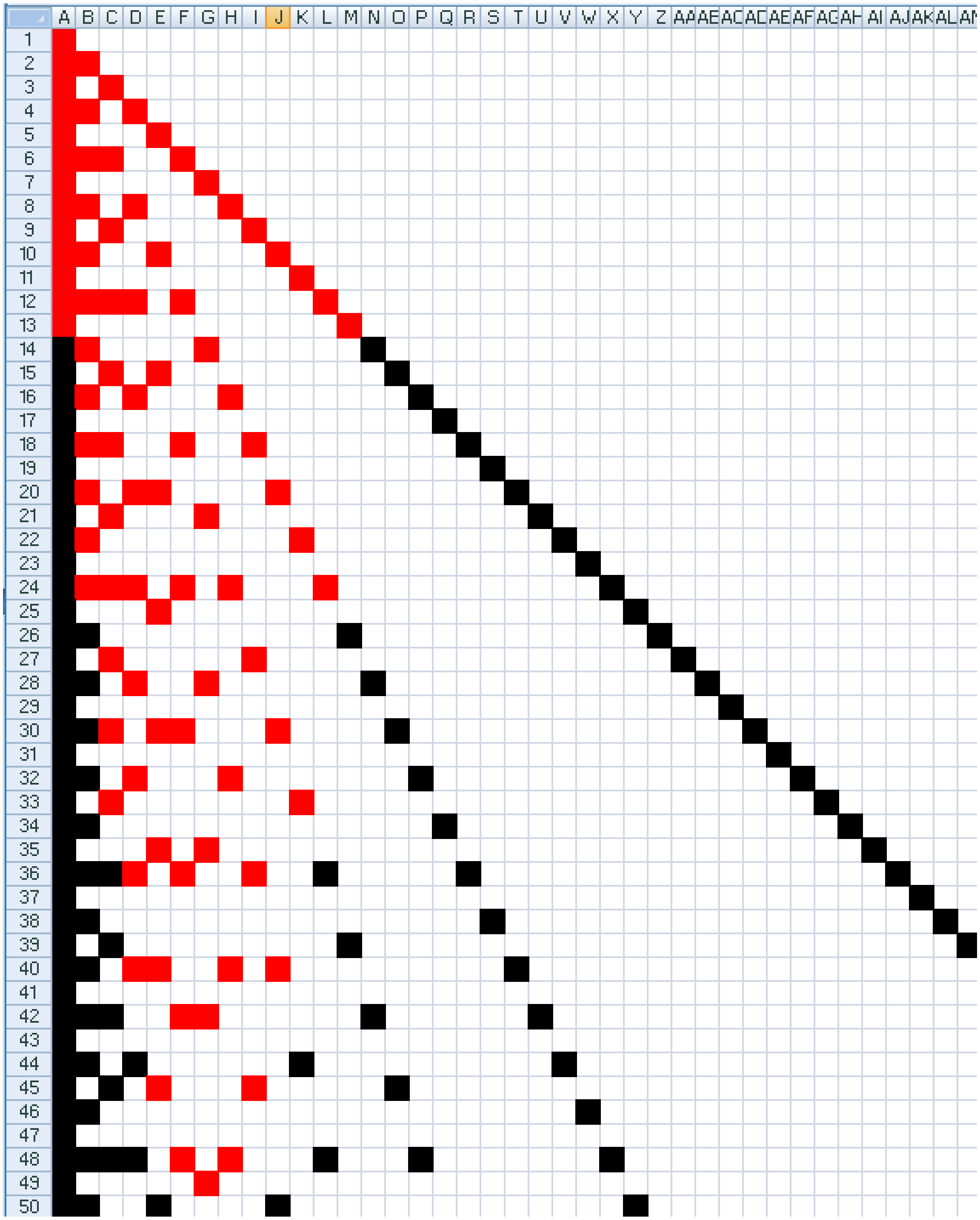
1. $(n + 0.5)^2 = (n-0)(n +0 + 1) + 0.25 +(0^2 + 0)$
2. $(n + 0.5)^2 = n(n + 1) + 0.25$, proven above.

When $a = 1$, you get:
1. $(n + 0.5)^2 = (n-1)(n +1 + 1) + 0.25 +(1^2 + 1)$
2. $(n + 0.5)^2 = (n-1)(n + 2) + 0.25 + 2$, proven above.

When $a = 2$, you get:
1. $(n + 0.5)^2 = (n-2)(n +2 + 1) + 0.25 +(2^2 + 2)$
2. $(n + 0.5)^2 = (n-2)(n + 3) +0.25 + 6$, as indicated above.

And so on.

Finally, we can ask the question: what happens when we combine both graphs? Are there any factors that are *not* covered by these two equations? Let us see. This time, I'll just mark red all the points covered earlier by the two sets of parabolas:



As you can see, this covers everything universally through row 13. So we have now found yet another way to represent the Factor Field (although this one has not been rigorously proven yet, at least by me). Namely, the Factor Field is represented by those whole number integer answers to the equations:

$n^2 = (n - a)(n + a) + a^2$

and

$(n + 0.5)^2 = (n - a)(n + a + 1) + 0.25 + (a^2 + a)$.

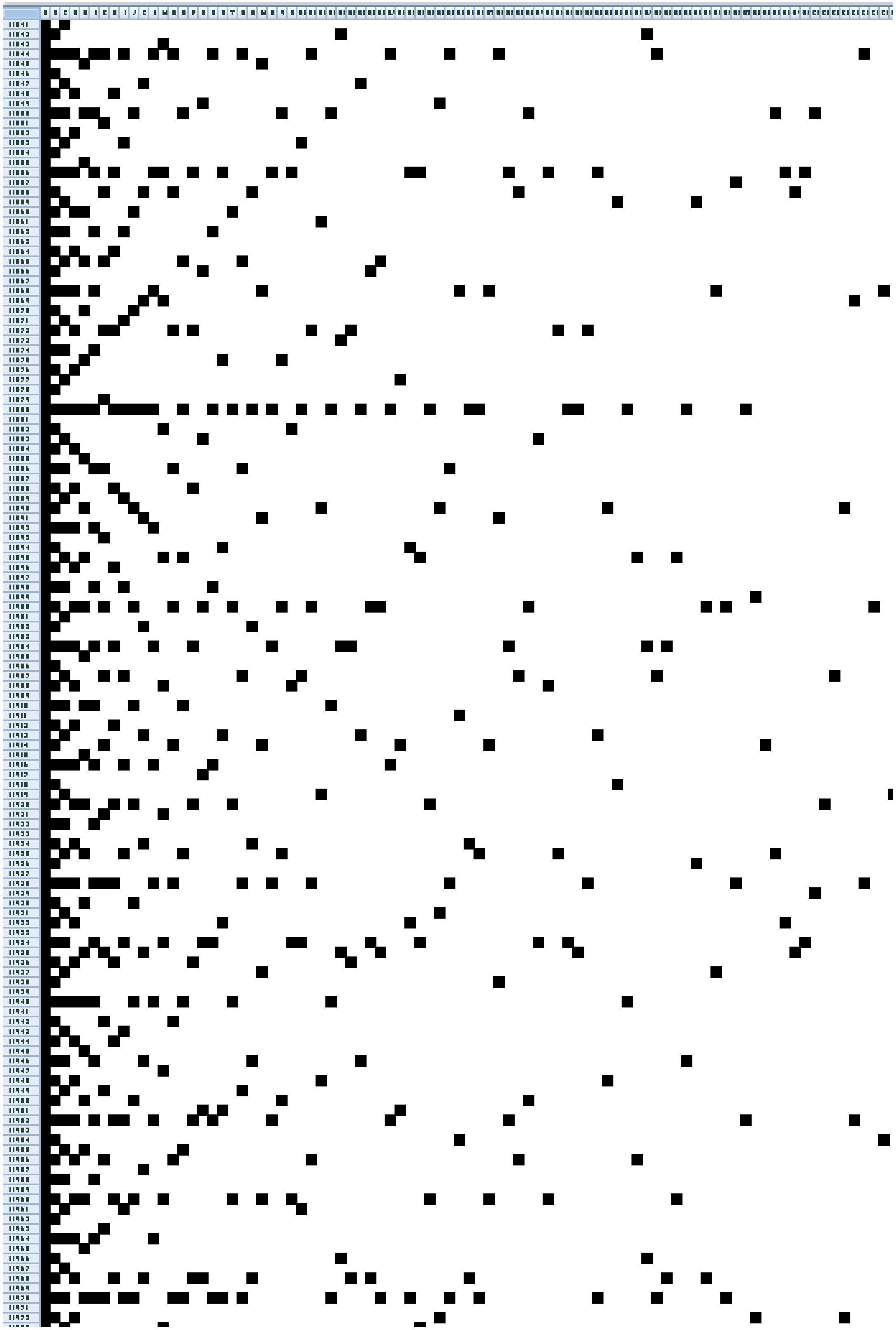
Now, concerning the ratio of "how fast" the Factor Field "fills up", we see the following:

1. 1^2 fills the graph through row 1.
2. 1.5^2 fills the graph through row 2.
3. 2^2 fills the graph through row 3.
4. 2.5^2 fills the graph through row 4.
5. 3^2 fills the graph through row 5.

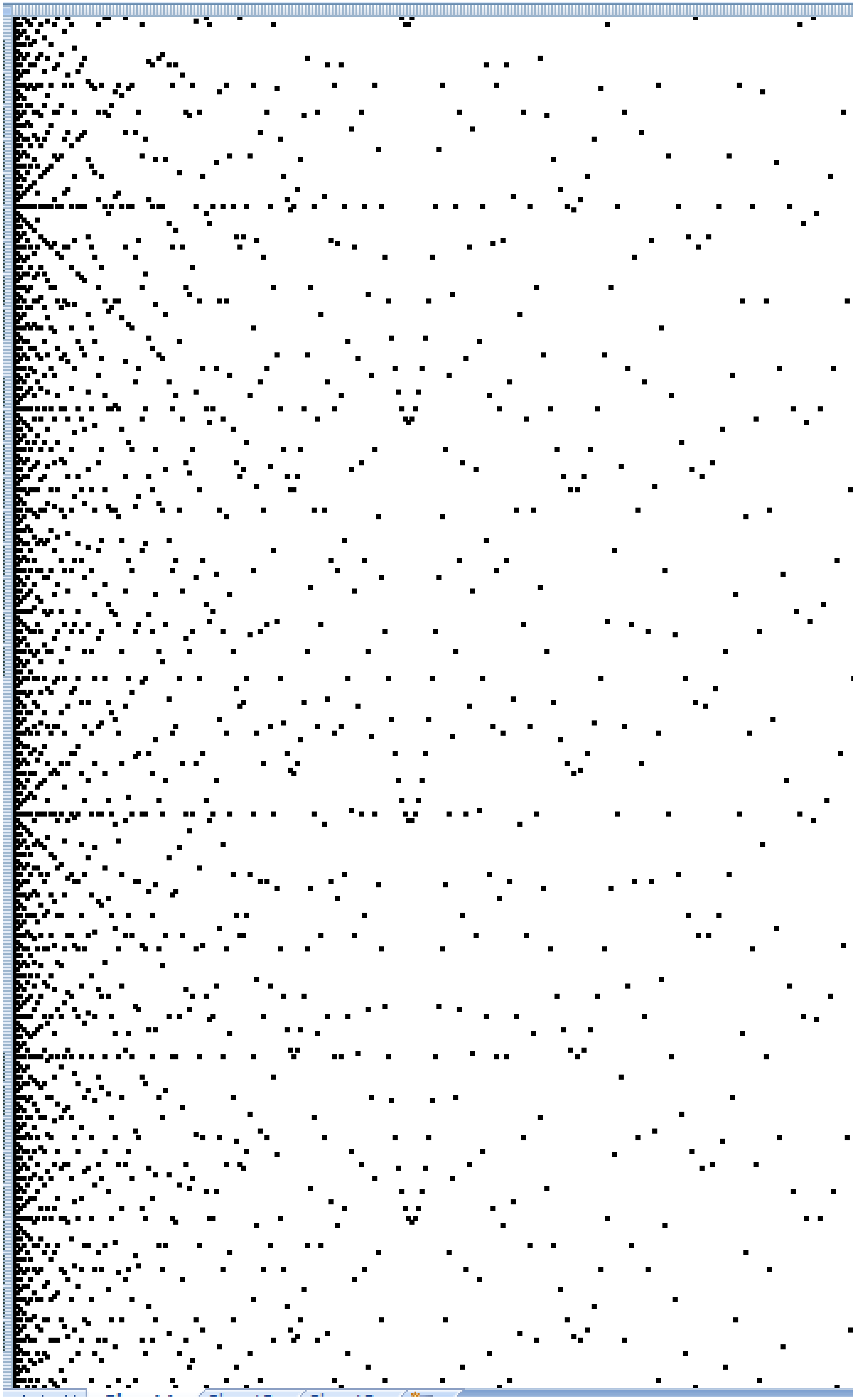
And so on. This is why we've seen that 7^2 fills the graph through row 13. So the general pattern is that for any number *n*, the Factor Field is "filled" through $2n - 1$ lines; where *n* is either a whole number or is a half. So, for instance, if $n = 7$, then it will fill up $(2 \times 7) - 1$, or 13 lines. If $n = 7.5$, then it will fill up $(2 \times 7.5) - 1$, or 14 lines. Etc.

Base-6 Math

One of the things that stands out to me when I look at the graph in more detail, especially as we move further down into the depths of the Factor Field, is how there is a repetition that "spikes" every six rows, with the occasional really massive ones. Look at this graph from around row 11841 (at the top) for example:



The large spike near the top 1/3 of the graph is at row 11880, for reference. Another good example begins at row 13804 and looks like this:



You can see several of the spikes there (as well as the parabolas we mentioned in the previous section).

The regularity of this 6-spike pattern makes me think that if we had discovered the Factor Field before learning how to count, we almost certainly would have developed a base-6 mathematical system. Nowhere is that more apparent then when we examine prime numbers.

As you'll recall from the beginning of this paper, a prime number is that which has a Factor Field Row Value of precisely 2. After 2 and 3, you can see the pattern emerge. The next numbers are 5 and 7, then 11 and 13. Then 17 and 19. Each one, so far, is on the pattern of $6n \pm 1$. However, the next sequence fails. While 23 is prime, 25 is not.

Still, as you examine the Factor Field it certainly *looks* like all the prime numbers greater than 3 occur on that $6n \pm 1$ pattern. But is there any way to prove it? It turns out, it's very simple to prove this theorem using base-6 math.

To prove it, we have to consider what happens when we multiply two numbers together. No matter how big the numbers are, the far right digit (the "ones" column) is fixed by a simple pattern. For example, in base-10 when multiplying by 5, the resulting number must end in either a 5 or a 0. When multiplying by 10, or any multiple of 10, it must end in 0. Anything multiplied by 2 must be even. And so on.

If we examine the table of endings in base-10, it would look like this:

Column x Row	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
6	0	6	2	8	4	0	6	2	8	4
7	0	7	4	1	8	5	2	9	6	3
8	0	8	6	4	2	0	8	6	4	2
9	0	9	8	7	6	5	4	3	2	1

In the same way, we can construct a table for base-6 endings. It would look like:

Column x Row	**0**	**1**	**2**	**3**	**4**	**5**
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

As you can see, the pattern from the 3 column replicates the pattern 5 has in base-10 math. That is, any number multiplied by the mid-point of the base you're in (at least for even numbers) will alternate endings between that number and 0.

Now, the important point, as far as we're concerned when it deals with prime numbers, is this: if prime numbers are based on a $6n \pm 1$ pattern in base-10, that would mean that in base-6 all prime numbers (greater than 3) would end in either a 1 or a 5. They would end in 1 if they are off the $6n + 1$ pattern, and they would end in 5 if they were off the $6n - 1$ pattern.

Examining the above, we can quickly see that if anything ends in an even number in base-6, it's divisible by 2 (just like in base-10 math). That means that numbers greater than 3 ending in 0, 2, or 4 cannot be prime. Furthermore, we know that any number ending in 3 or 0 is divisible by 3, just as any number ending in 5 or 0 is divisible by 5 in base-10. So that means that numbers greater than 3 ending in 0 or 3 likewise cannot be prime. Thus, we have excluded all numbers ending in 0, 2, 3, or 4, greater than 3, from being prime. That leaves us only numbers ending in 1 or 5. Thus, the $6n \pm 1$ pattern is proven valid. Although not all numbers that fit the $6n \pm 1$ pattern are prime, all prime numbers greater than 3 do fit that pattern.

For ease of reference, I will call these numbers **P** numbers, because they are possibly prime. I will define P^+ as a number that end in 1, or corresponds to the $6n + 1$ pattern in base-10, and P^- for those that end in 5, or corresponds to the $6n - 1$ pattern.

From the table, we can see that:

1. P^+ multiplied by another P^+ will give a P^- results.
2. P^- multiplied by another P^- will give a P^- results.
3. P^+ multiplied by a P^- will give a P^+ results.

In other words, for base-6 numbers:

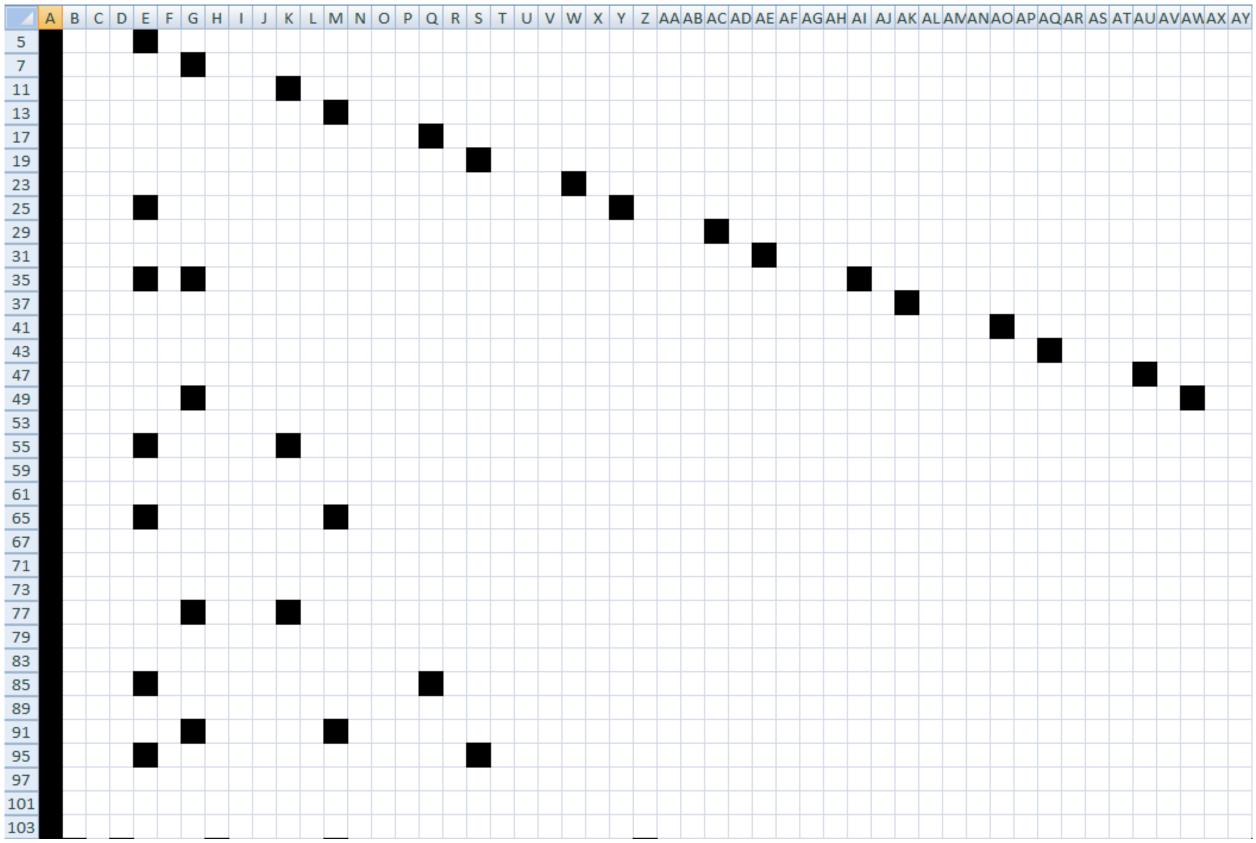
1. 1 x 1 ends in 1
2. 5 x 5 ends in 1
3. 5 x 1 / 1 x 5 ends in 5

We can further conclude, regarding P numbers, that if a P number turns out not to be prime itself, then the only factors it can possibly have are other P numbers. This follows because, if you look at the chart again, only numbers ending in 1 or 5 can be multiplied together to get a number ending in 1 or 5.

Now the consequences of this insight are vast. When you are checking to see if a number is prime, *you only need to consider P numbers*! All other numbers are irrelevant. This means that you can ignore 2/3s of the numbers that exist right off the bat. And we shall now put that fact to use.

Prime Numbers

Given that we can ignore all but the P numbers when looking at primes, we can condense our Factor Field immensely (when focusing on primes) by hiding some rows. For instance, just looking at the P number rows from 1 to 101 gives us this:

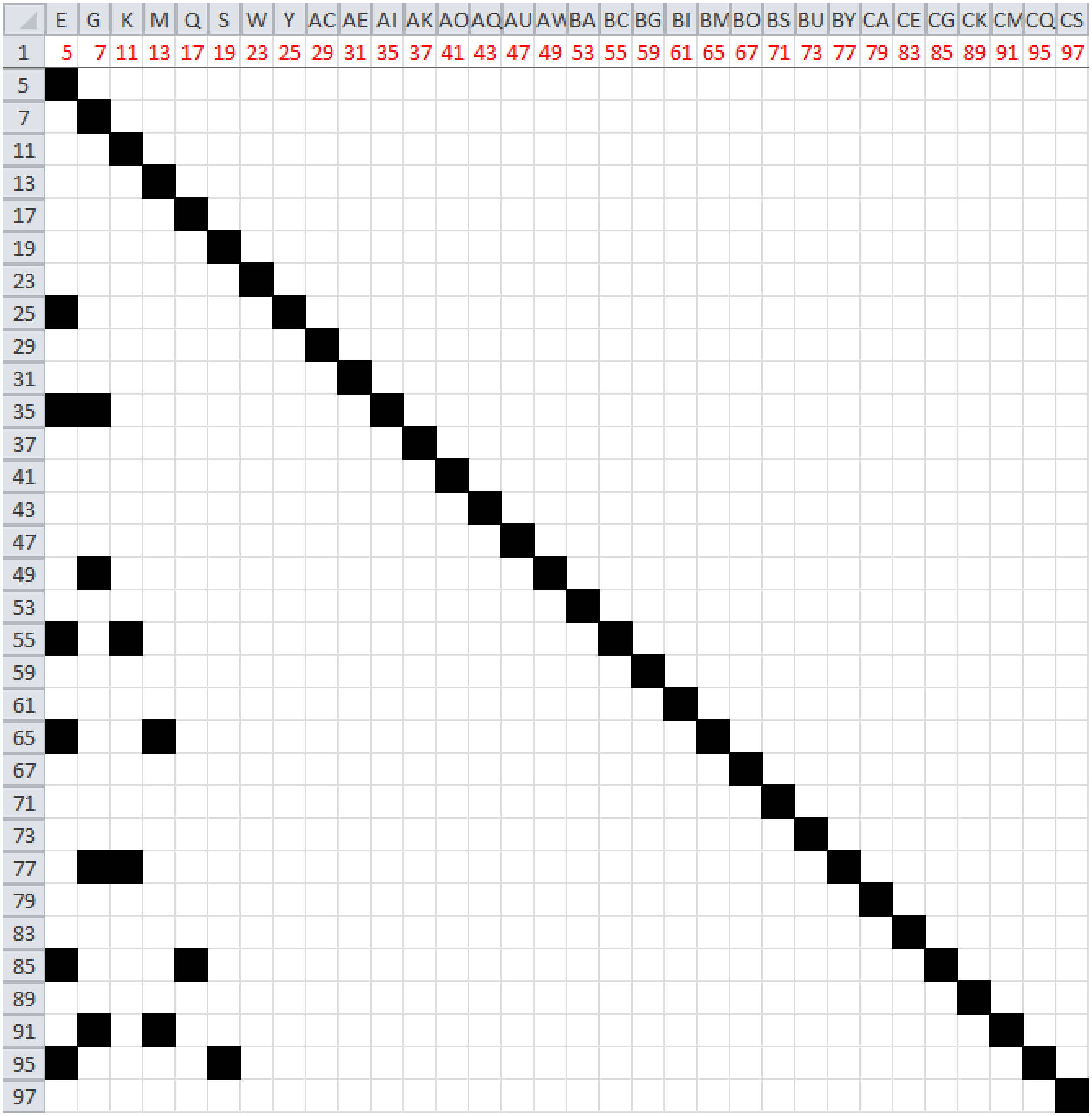


Now this already shows us some interesting patterns. For one thing, if we ignore the far left-column and the first dashed line at the top (both of which represent 1 x the P number), wherever there is a colored cell, that row represents a number that is *not* a prime. So, in essence, the first relevant dot is at for 25. And 25 is not prime. We also see that 35 has two dots; and 35 is not prime either. Likewise, 49, 55, 65, 77, 85, 91, and 95.

Now this makes perfect sense when you think about it, because this is just displaying the rows we have already seen before, and we know that a prime number only has two factors, so these rows with extra dots are the ones that add up to more than 2 factors per row.

But I had an insight while looking at this. The vertical columns have a pattern. The first one (excluding the ones column) is column E, representing 5. As we look down, we see that we have a dot followed by 6 empty spaces. Then there's a dot followed by 2 empty spaces. Then the pattern repeats: dot, 6 empty spaces, dot, 2 empty spaces.

Next to that is the 7 column (here represented by G). You have a skip of 1. Then there's a dot, 8 empty spaces, a dot, then 4 empty spaces, and the repeat.



The above graph removes the extraneous columns so we can more easily see the pattern emerge. Specifically, where there is a dot (excluding the diagonal line, which represents the x 1 aspect of the pattern), the number represented by the row is *not* prime. The pattern is as follows:

5 = 0 blank, square, 6 empty, square, 2 empty. Repeat
7 = 1 blank, square, 8 empty, square, 4 empty. Repeat
11 = 2 blank, square, 14 empty, square, 6 empty. Repeat
13 = 3 blank, square, 16 empty, square, 8 empty. Repeat
17 = 4 blank, square, 22 empty, square, 10 empty. Repeat
19 = 5 blank, square, 24 empty, square, 12 empty. Repeat

Etc.

To put it in other words, the pattern is as follows: increase the initial blank by 1 each time. The second "empty" section increases by 2 each time. The distance in the first "empty" space increases by 2 between the pairs of values (6 -> 8; 14 ->16, etc.).

What explains this pattern? Remember, these are all P numbers, and a P number is in the pattern of $6n \pm 1$. So let's substitute that in for the first value of 5, using the $6n - 1$ portion of $6n \pm 1$:

$5 \times (6n - 1)$

Now, we can quickly do math and solve this one:

$5 \times (6n - 1) = 30n - 5$

Let's add some test values for n.

When $n = 1$, the solution is 25.
When $n = 2$, the solution is 55.
When $n = 3$, the solution is 85.

When we look at the difference in the values for the solution, we see that the difference in the solution for subsequent values of n is exactly 30. This comes from the $30n$ portion of the equation, so that's to be expected. Furthermore, the equation is offset by -5, so it begins at 25 and moves forward by 30 each time.

Now consider the $6n + 1$ values:

$5 \times (6n + 1) = 30n + 5$.

The only difference is that instead of subtracting 5, we add 5 to the value. So:

When $n = 1$, the solution is 35.
When $n = 2$, the solution is 65.
When $n = 3$, the solution is 95.

Once again, the difference between subsequent values of n is exactly 30. This time it's offset by +5, though, and that means the difference between the solutions for $6n - 1$ and $6n + 1$ is 10. (I.e., $35 - 25 = 10$; $65 - 55 = 10$, etc.)

This is all well and good, but let's consider what we get when we examine $7 \times 6n \pm 1$. First, we begin once more with $6n - 1$, as follows:

$7 \times (6n - 1) = 42n - 7$.

When $n = 1$, the solution is 35.
When $n = 2$, the solution is 77.
When $n = 3$, the solution is 119.

Here ,the difference is 42 each time ($42n$), offset by – 7 so it begins at 35. Likewise, we have:

$7 \times (6n + 1) = 42n + 7$.

When $n = 1$, the solution is 49.
When $n = 2$, the solution is 91.
When $n = 3$, the solution is 133.

Again ,the difference is 42, offset by +7 so it begins at 49. So each sequence is spaced out by 42 each time, and the difference between the two sets of solutions is once 14, or 2 x 7.

Now the final consideration. Let us compare the numbers for 11 to what we got for 5, and 13 to 7. First, the characteristics of 11 are as follows:

$11 \times (6n - 1) = 66n - 11$. Answers for $n = 1$ to 3 are: 55, 121, 187. The difference between each is 66.
$11 \times (6n + 1) = 66n + 11$. Answers for $n = 1$ to 3 are: 77, 143, 209. The difference is again 66. Offset between the two values is 22, which is 2 x 11.

When we compare these values to 5, we see the first that 11 = 5 + 6. Then we can note that $30n$ increases to $66n$, which means it increases by 36, or 6^2. The amount we add and subtract at the end increases by 6 too, once again from 5 to 11.

Now the characteristics of 13 are:

$13 \times (6n - 1) = 78n - 13$. $n = 1$ to 3: 65, 143, 221.
$13 \times (6n + 1) = 78n + 13$. $n = 1$ to 3: 91, 169, 247. Again, the difference is 13, and the offset is 26, or 2 x 13.

When we compare this to 7 we get the expected results. First, 13 = 7 + 6. $78n - 42n = 36n$, as before.

So, what can we determine of the general pattern. All the patterns are as follows:

1. $(6a - 1)(6n - 1) = 36an - 6a - 6n + 1$
2. $(6a + 1)(6n - 1) = 36an - 6a + 6n - 1$
3. $(6a - 1)(6n + 1) = 36an + 6a - 6n - 1$
4. $(6a + 1)(6n + 1) = 36an + 6a + 6n + 1$

To demonstrate how this works out:
When $a = 1$:

1. $(6 - 1)(6n - 1) = (36 \times 1 \times n) - 6 - 6n + 1$
   $5(6n - 1) = 36n - 6n - 5$
   $30n - 5 = 30n - 5$ (as shown above)

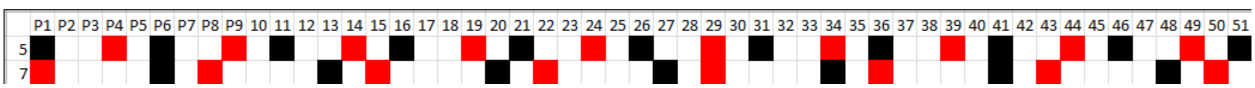
2. $(6 + 1)(6n - 1) = (36 \times 1 \times n) - 6 + 6n - 1$
   $7(6n - 1) = 36n - 6 + 6n - 1$
   $42n - 7 = 42n - 7$ (as shown above)

3. (6 – 1)(6*n* + 1) = (36 x 1 x *n*) + 6 – 6*n* – 1
   30*n* + 5 = 36*n* + 6 – 6*n* – 1
   30*n* + 5 = 30*n* + 5 (as shown above)

4. (6 + 1)(6*n* + 1) = (36 x 1 x *n*) + 6 + 6*n* + 1
   7(6*n* + 1) = 36*n* + 6 + 6*n* + 1
   42*n* + 7 = 42*n* + 7 (as shown above)

So, as *a* and *n* both tend to infinity, these sequences determine where numbers are *not* prime.

But if we keep in mind what we've learned about P numbers, we can actually reorganize this a bit and extract more useful information here. To demonstrate this, I'm going to rotate the graph (to keep it from becoming a gigantically lengthy mess on the page) and use a color code. For the following, we will use a black dot to represent a P^- number (which corresponds to 6*n* - 1 in base-10 math, or a number ending in 5 in base-6 math), and a red dot to represent a P^+ number (which corresponds to 6*n* + 1 in base-10 math, or a number ending in 1 in base-6 math). We can then "compress" the columns by making them represent P numbers as a whole (i.e., both P^+ and P^- number together). For our first two rows, we generate this graph:



To read this, we know 5 and 7 are the results of 6*n* ± 1 when *n* = 1. So, the two horizontal rows represent those values. The P1 column likewise represents 6*n* ± 1 when *n* = 1. So the columns are "compressed" but the rows are not.

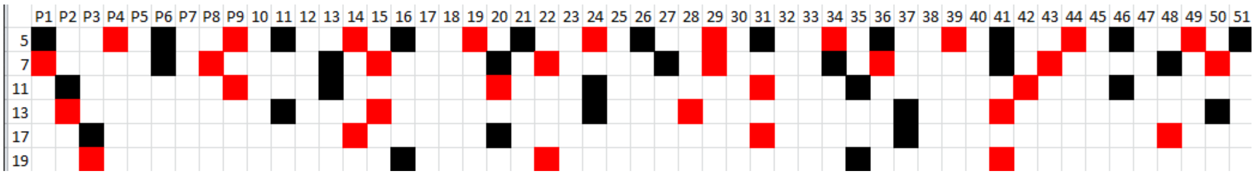
Looking at the first column, we see a black dot for 5 and a red dot for 7. That's because the first column, P1, represents 6*n* ± 1 when *n* = 1, or the numbers 5 and 7 themselves. In other words, the first cell at 5 is colored because it represents 5 x 1, and since that is a P^- number we see a black cell. Similarly, the first cell at 7 is colored red because it represents 7 x 1, and that is a P^+ number.

We next see on 5's row a red dot at P4. P4 represents 6*n* ± 1 when *n* = 4, or the values of 23 and 25. The red dot is thus equivalent to 5 x 5, or 25, which is a P^+ number, specifically (6 x 4) + 1, the + 1 making it red.

The final column I'll examine here for descriptive purposes is P6, which is equivalent to 36 ± 1, or 35 and 37. Here, we see that both the 5 row and the 7 row have black dots. That's because 35 is 5 x 7, and 35 is a P^- number, hence it's black color.

This structure gives us the beginning of a prime number "sieve". Wherever a dot appears, it blocks a prime number from that location. But remember that it blocks it in a specific way. A black dot blocks the P^- number and a red dot blocks the P^+ number. At P6, 5 and 7 both block *the same* number from being prime: 35. However, when there is a black and red together in the same column, it blocks the entire P value from being prime (for examples, see P34 and P36). The only exception to this is the P1 column, since that represents 5 x 1 and 7 x 1.

We can continue by adding in the next two rows, represented by 11 and 13, as well as the one after that representing 17 and 19:



By comparing the pair of rows, we can see a pattern emerging. So what is the pattern here?

There are actually two patterns. One for the P^- values, and one for the P^+ values.
For the P^- values:

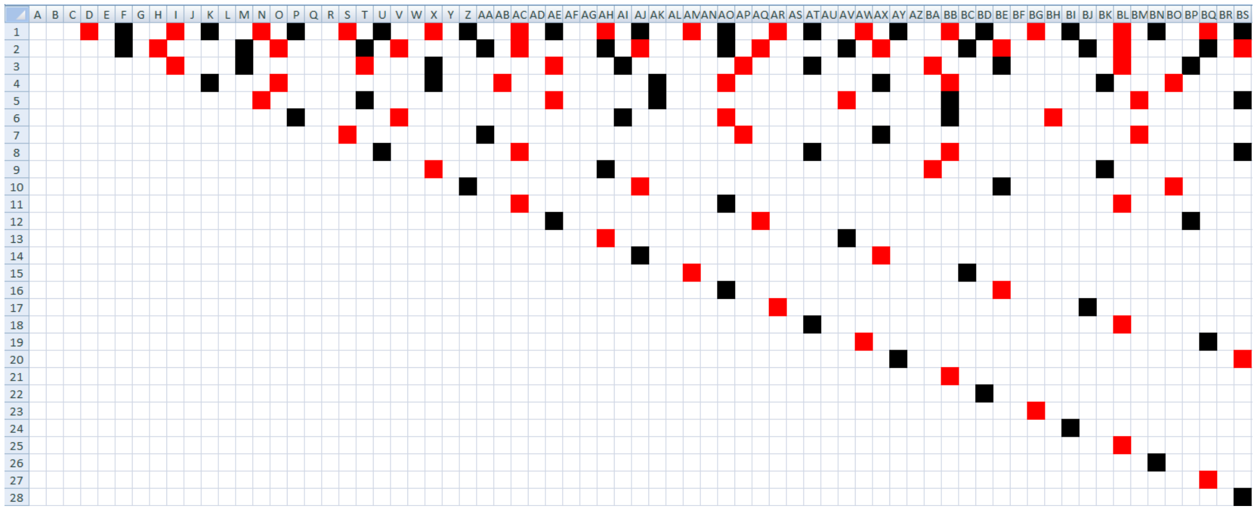
1. Skip *x* columns for our initial skip. This begins with $x = 0$ for rows 5 and 7, then becomes $x = 1$ for 11 and 13, and $x = 2$ for 17 and 19, etc.
2. Fill in a black cell.
3. Skip $4x – 2$ columns.
4. Fill in a red cell.
5. Skip $2x – 1$ columns.
6. Repeat from step 2.

For the P^+ values:

1. Skip *x* columns again, beginning with x = 0 (as stated above).
2. Fill in a red cell.
3. Skip $4x$ columns.
4. Fill in a black cell.
5. Skip $2x – 1$ columns.
6. Repeat from step 2.

The only difference between the two patterns is that the filling of black and red cells is flipped, and the skip in step 3 is different. Other than that, the two patterns are identical.

The initial “stacked” values, representing the x 1 values, can be ignored for practical purposes, and as usual I’ve made a VBScript that will do this in Excel:



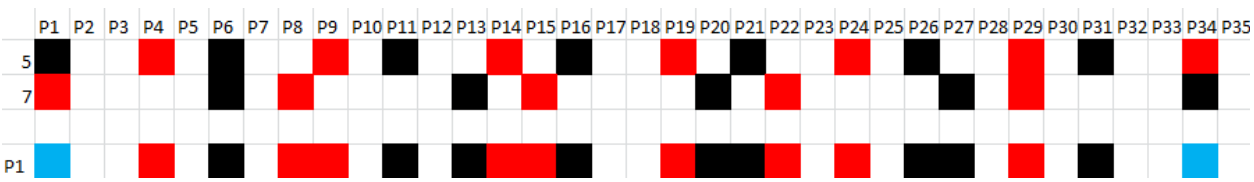
```
set objExcel = CreateObject("Excel.Application")
set objWorkbook = objExcel.Workbooks.Add()
set objWorksheet = objWorkbook.Worksheets(1)
objExcel.Visible = True

' Use these values for a practical demonstration
MaxWidth = 100
MaxDepth = 100

for n = 1 to MaxDepth
     row = (n * 2) - 1
     i = n 'Initial skip

' Create the P- side
for q = 1 to MaxWidth
     i = i + ((4 * n) - 2) + 1
     If i > MaxWidth Then
          Exit for
     End If

     objExcel.Cells(row, i).Interior.ColorIndex = 3
     i = i + ((2 * n) - 1) + 1

     If i > MaxWidth Then
          Exit for
     End If

     objExcel.Cells(row, i).Interior.ColorIndex = 1
next

     row = (n * 2)
     i = n ' Initial skip

' Create the P+ side
For q = 1 to MaxWidth
     i = i + (4 * n) + 1

     If i > MaxWidth Then
          Exit for
```

```
        End If

        objExcel.Cells(row, i).Interior.ColorIndex = 1
        i = i + ((2 * n) - 1) + 1

        If i > MaxWidth Then
             Exit for
        End If

        objExcel.Cells(row, i).Interior.ColorIndex = 3
        next
next

wscript.Echo "Done."
```

If you change the MaxWidth and MaxDepth values, you can change the size of the graph produced. As long as you have sufficient rows and columns, you'll be able to see what P values have twin primes, which have only a single prime, and which have no primes at all. All you need to do is go to the column and look for colored cells. If all the cells are white, you have a twin prime at that particular *n* at that P value. If you have one or more red cells, but no black cells, then the P^- value is prime but the P^+ is not. If you have one or more black cells, but no red cells, then the P^+ value is prime, but the P^- is not. Finally, if the column contains one or more of both red and black cells, then there are no primes at that P number at all.

If we add in a third color (in this case, blue), we can now compress the pairs of rows in the same way that we compressed the columns. For this, a black dot will represent with the P^- value is blocked from being prime, a red will represent when the P^+ value is blocked, and a blue will represent when *both* are blocked.

If we sum the first two rows (the 5 and 7) together with those rules, we get the following:



To fully represent the pattern, I've included the P1 value again, which was excluded in the Excel graph created using the VBScript above. The pattern begins again at P36, since it is looping every 35 cells. This makes sense, since 5 x 7 = 35.

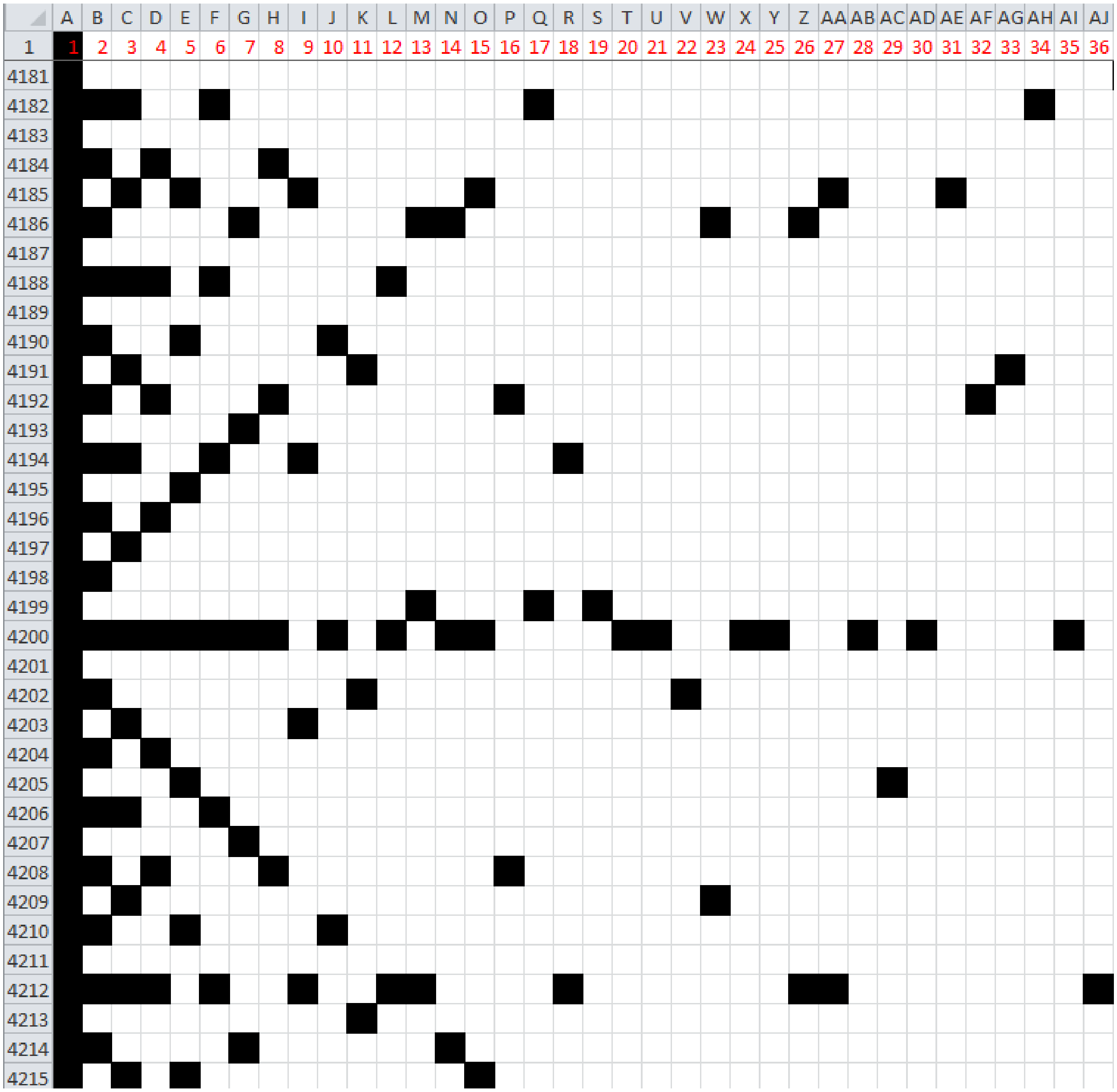
Looking at this pattern, we can see symmetry in it. Ignoring the P35 column for now, we see that the pattern begins and ends with a blue square. Then there are two blanks. At P4, there is a red square, which shows up as black at P31. At P6 a black square, mirrored as red at P29. Then two red squares at P8 – P9, mirrored by two black squares at P26 – P27, etc.

The same mirroring exists when we look at 11 and 13. Now, however, the pattern is even more spread out since it doesn't repeat for 143 columns (11 x 13 = 143). It is also shifted over 1 at the beginning, so it actually repeats on column P145. Additionally, the final blue dot is actually on cell 142 (which is 141 steps in once you account for the offset). This means that while the gap between the last blue cell and the repeated sequence for the 5 and 7 lines was only one cell, here it is a gap of three cells.

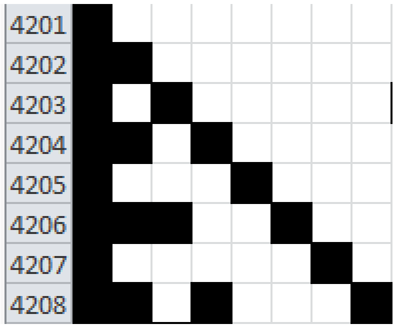
While there are lots of similarities and patterns that we can find here, these patterns are not as straight-forward as the previous patterns we've uncovered. Consequently, while more investigation might prove useful here, I am going to move on to the next topic for now.

Reflections

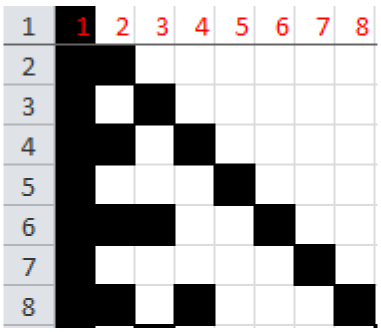
I want to touch on one final aspect of the Factor Field, and that's the aspect of reflections. As I mentioned before, there is a spike that occurs every 6 rows. Sometimes, there are even bigger spikes than usual. Take a look at cell 4200, for instance, here:



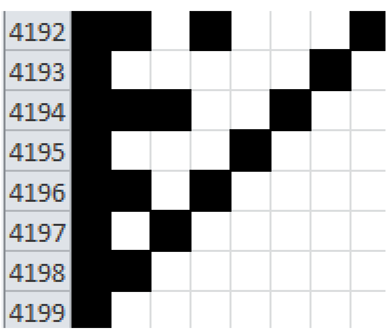
Now, let's see what happens when we "zoom" in for a moment. Let's look in particular at this point:



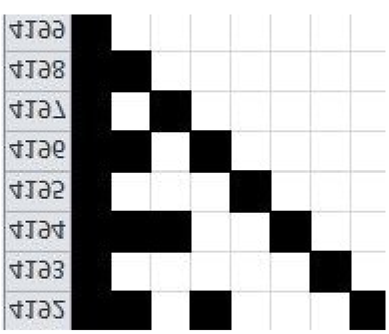
Does it look familiar? Compare it to this graph:



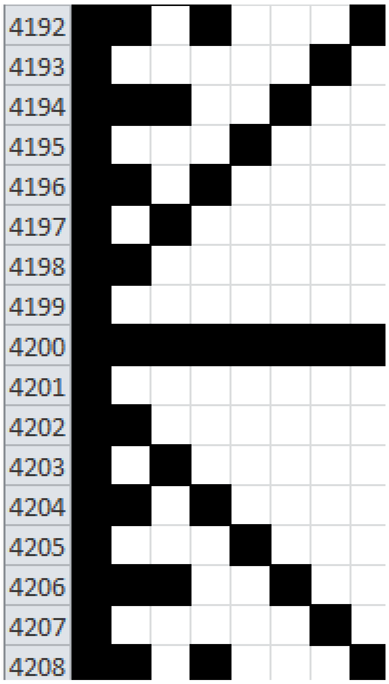
As you can see, this is exactly the same as the beginning of the Factor Field. Furthermore, if we look at the part immediately above row 4200, we see this:



If we rotate this picture 180 degrees and then flip on the horizontal axis, we get this:



In other words, the portion above 4200 reflects what occurs below 4200. Of course, it only does this for 8 columns before the pattern "breaks", but during that time we get this:



The reason this works is because 1, 2, 3, 4, 5, 6, 7, and 8 are all factors of 4200. And because of that, we learn an important fact. When we get a long string of factors, then the Factor Field will "recapitulate" below the number with that many factors, and it will "reflect" above that point. And this makes sense when you think about it, because the Factor Field is developed from a pattern of numbers, and that pattern must continue throughout.

At 4200, for instance, we know that 2 is a factor. Therefore, it is impossible for 4201 or 4199 to be divisible by 2, since the next number that possibly *could* have 2 as a factor is 2 greater than 4200. And the next previous number that possibly *could* have 2 as a factor is 4198. And not only *could* it have it, it *must* have that factor there.

The same thing is true for 3. Since 3 is a factor of 4200, then next number that could possibly have 3 as a factor is 4203. The next previous number was 4197. Etc.

There are two things that cash out from this. One is more obvious than the other. That more obvious one is: since the Factor Field itself is being replicated, in a miniature form, and since the leading edge of the factor field includes the diagonal arm that goes from (1,1) down, then where this is replicated there will be a section that contains two diagonal arms. One over here, and the other being the original Factor Field arm.

While that paragraph is a mouthful, the upshot is that when viewing a number like 4200, then aside from the possibility of 4199 and 4201 being prime, it is impossible for there to be any more prime numbers until the diagonal arm "runs out". I will prove this shortly using more conventional math, but let me first state that this means that we can construct arbitrarily long "droughts" where it is impossible for there to be prime numbers.

The proof of this is based on the observation I just made above regarding how factors work. Let us construct a number, N, where $N = 1 \times 2 \times 3 \times 4 \ldots \times n$. This n can be arbitrarily large, so we can have literally thousands of factors involved. N will consequently be a massive number, but we can still conclude the following:

$N \pm 2$ cannot be prime since 2 is a factor of N so $N \pm 2$ is also divisible by 2.
$N \pm 3$ cannot be prime since 3 is a factor of N so $N \pm 3$ is also divisible by 3.
$N \pm 4$ cannot be prime since 4 is a factor of N so $N \pm 4$ is also divisible by 4.
.
.
.
$N \pm n$ cannot be prime since n is a factor of N so $N \pm n$ is also divisible by n.

This means that with the exception of $N \pm 1$, it is *impossible* for there to be any prime numbers between $N - n$ and $N + n$. Also note that this does not guarantee that either part of $N \pm 1$ is itself prime either.

Time to Get Controversial!

But what about the second, less obvious aspect? For that, we'll need to bend the rules a bit. We're going to violate a rule of mathematics, but in such a way as to draw an interesting point that, I believe, has some bearing on reality, even if it's not fully in compliance with traditional "rules."

Let's take N in the above argument all the way to infinity. Let us do this by defining a special kind of infinity. We are going to take $\infty!$ (That is, infinity factorial.) From that, let us define a number, Z, as: $Z = \infty! + 1$.

Now, obviously we cannot literally add 1 to infinity, because that just gives us infinity again. But given the way we constructed infinity here, let's actually think about this. What would Z be if it were actually possible to create Z? What properties would it have? Well…

We know that Z cannot be divisible by 2, because $Z - 1$ is divisible by 2.
We know that Z cannot be divisible by 3, because $Z - 1$ is divisible by 3.
We know that Z cannot be divisible by 4, because $Z - 1$ is divisible by 4.
.
.
.
Etc.

So, literally the only number that Z is divisible by is…1. But the only number that exists that is divisible only by 1 is the number 1 itself. Thus, $Z = 1$.

Furthermore we can look at $Z + 1$ (which is the same as $\infty! + 2$).

We know that $Z + 1$ *is* divisible by 2, because $Z - 1$ is divisible by 2, so $Z + 1$ is also divisible by 2.

But, $Z + 1$ is not divisible by any other number greater than 2, because $Z - 1$ is divisible by each of those numbers. Thus, $Z + 1$ is divisible by only 1 and 2. The only number that is divisible by only 1 and 2 is the number 2 itself.

Therefore, $Z + 1 = 2$.

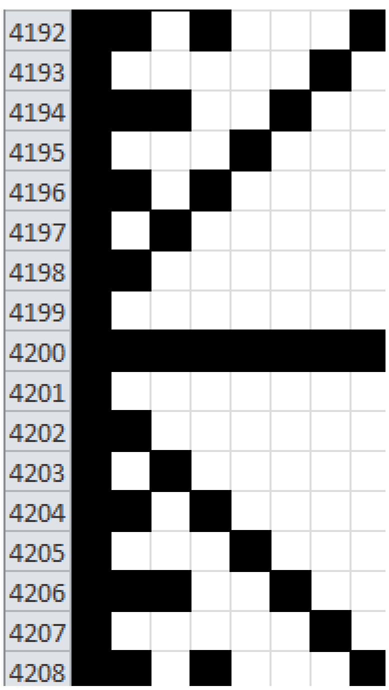
Therefore, $Z = 1$, yet again.

But here's the kicker. If $Z = 1$, then that is saying $\infty! + 1 = 1$.

If $\infty! + 1 = 1$, then $\infty! = 0$.

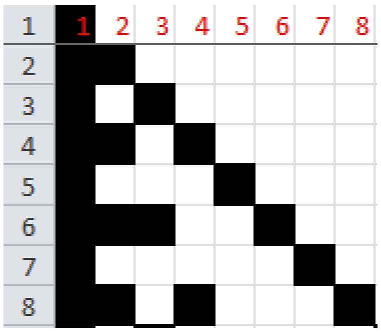
That's right. If you multiply all the numbers together, you end up with a result of 0.

Now, before you just dismiss this as an arbitrary fluke of trying to do math with infinities, let me just point out that the Factor Field itself lends itself to this interpretation when you think of the portions of the Factor Field that were "reflected" and "recapitulated" in our previous section. Look again at row 4200:



If the bar formed at row 4200 actually extended an infinite distance, then an exact duplicate of the Factor Field would form beneath it.

Additionally, look at the beginning of the Factor Field again:



If we were to extend the Factor Field up to include 0 and then -1, -2, etc. what would it look like? Row 0 would obviously have a dot at column 1, since the 1s column is always filled. But it would also have a dot at 2, because 0 is the next factor above 2 that is divisible by 2. Likewise with 3, 4, 5, etc.

In other words, row 0 would have a line of infinite length. Then, the negative numbers would reflect the positive numbers across that axis.

Now, it is certainly the case that 0 is divisible by 1, as 0/1 = 0. It's also divisible by 2, since 0/2 is 0. So, 0 is divisible by all numbers, and therefore it wouldn't be a violation of the rules of math to extend the factor field through to 0.

The problem is that while 0 would have every factor, so too, would ∞! be divisible by all numbers. In other words, a Factor Field extended to include 0 would be indistinguishable from a Factor Field extended to include ∞!.

Again, I realize that this "works" because we are ignoring some rules of math involving infinities. But I ask, is this really a problem? After all, we all know that there is no actual value for the square root of negative 1, yet we still treat *i* as an important and useful value. It's *imaginary* and yet valid to use. Furthermore, the process I used to determine that ∞! = 0 is essentially the same process that Cantor used for his diagonals, which proved that an infinite list of numbers was still *missing* an infinite amount of entries. In other words, these processes still provided useful information—some golden nugget of value.

Could it be the same for the claim that ∞! = 0?

Open Questions

Given everything listed above, I have a few open questions I've yet to resolve. In one sense, I want to just go ahead and investigate these myself, but ultimately I have to actually submit this at *some* point or it will never get done. Investigating some of these will be easier than others, but in the end they will almost certainly lead to even more open questions.

For now, the questions I have are as follows.

1) Given the "reflection" that occurs (such as at cell 4200), there will be stretches of the Factor Field that will reflect everything we've thus far seen. That will include the structures that we see formed from the perfect squares of numbers. In other words, we know that there is a "chevron" at (3,9), (4,16), etc. What is the mathematical formula for these structures when they are reflected?

2) Closely related to (1), if we extend the Factor Field through zero and then into negative numbers, since squares are positive values, would we just express the negative values in terms of *i* or is there a better method of examining these negative numbers?

3) While it is the case that ∞! would have all the factors of every number, the way it is defined (that is: 1 x 2 x 3 x …) would not be the smallest possible value that has all those factors. For instance, looking at the aforementioned cell 4200 again. It has the first 8 numbers as factors, yet 8! = 40,320. In fact, 4,200 not the smallest value that has all 8 factors. That distinction goes to cell 840, since 840 = 2^3 x 3 x 5 x 7. This means that 840 is divisible by 8 (2^3), 7, 6 (2 x 3), 5, 4 (2^2), 3, 2, and of course 1. Since this pattern recreates every 840 rows, this means that there are 48 instances where this feature is in the Factor Field before we reach the value of 8!. Given that, we know that in some sense there is an infinity that is "smaller" than ∞! which would have every integer as factors. Is there a way to tease out what that definition of infinity would look like?

Final Thoughts

At this point, I will offer some final thoughts on the Factor Field. I state again that I am not a mathematician. I am a computer programmer who gets to function in a world formed by nested loops. It is no surprise to me that I find nested loops in mathematics too. The number line is itself a loop containing an infinite number of other loops, and so on.

What is amazing to me is not that I have discovered new math, since I believe all the conclusions I've derived have probably already been determined elsewhere. When I first thought I had come up with a unique concept of prime numbers being patterned after $6n \pm 1$, you can imagine my disappointment when I read Wikipedia and discovered this "new fact" had been known for hundreds of years already. So in that sense, as far as I know I haven't discovered any new insights with the Factor Field.

But what it has done is allowed an amateur—someone who hates calculating even though he loves number theory—to be able to discover deep mathematical insights just by *glancing* at a graph. I never would have surmised that prime numbers are based off of $6n \pm 1$ just by looking at a list of prime numbers (although there are obviously mathematicians who can do just that).

The Factor Field is therefore useful for opening up avenues of thinking about mathematics for those who are not computationally inclined. I suspect that it will work really well for those who like geometry over algebra. The Factor Field enabled me to use my scientific reasoning: observe, hypothesize, re-evaluate, form a theory, then try to prove it with mathematical rigor. From that, I could deduce how squares form all factors, and how even the 0.5 square is important. I discovered deep insights about the nature of prime numbers. I even discovered what could be an important point about infinity and its relation to zero. This would have been much more difficult had it not been for the existence of the Factor Field.

Thus, I would argue that even if there isn't a single new thing learned, the Factor Field provides an easier way of understanding concepts that make it worthwhile to investigate further. For that reason, I want to get this data out to the public at large. If I, a mere amateur, could come up with the concepts that I have come up with just by looking at this structure, imagine what heights of glory could be attained by someone who knew what they were doing with mathematics!